Supplementary Material

Simultaneous Bayesian Estimation of Excitatory and Inhibitory Synaptic Conductances by Exploiting Multiple Recorded Trials

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Contents

Appendix I. Problem Formulation

The dynamical system representing the dynamics of each recorded membrane potential (MP) is defined as follows:

$$\begin{cases} \mathbf{x}^{\mathrm{i}}(t+1) = f(\mathbf{x}^{\mathrm{i}}(t)) + \mathbf{v}(t) \\ y^{i}(t+1) = \hat{C}\mathbf{x}^{\mathrm{i}}(t+1) + \sqrt{dt}\sigma_{y}\varepsilon^{i}(t) \end{cases}, \tag{A.1}$$

where $\boldsymbol{v}(t)=[w(t), N_E(t), N_I(t)]^{\mathrm{H}}$, index i corresponds to the i^{th} Kalman filter that is derived for the i^{th} trial, and

$$\begin{aligned} &\mathbf{x}^{i}(t) = [V^{i}(t), g_{E}^{i}(t), g_{I}^{i}(t)]^{H} \\ &\mathrm{X} = [\mathbf{x}^{1}(0:T),...,\mathbf{x}^{i}(0:T),...,\mathbf{x}^{L}(0:T)] \end{aligned}. \tag{A.2}$$

Both excitatory and inhibitory synaptic inputs (SIs) are approximated by a Gaussian distribution:

$$\begin{aligned} &\boldsymbol{N_E}(t) \approx Gaussian\,(\boldsymbol{N_E}(t); \boldsymbol{\mu}_{NE}(t), \boldsymbol{\Gamma}_{NE}(t)), \\ &\boldsymbol{N_I}(t) \approx Gaussian\,(\boldsymbol{N_I}(t); \boldsymbol{\mu}_{NI}(t), \boldsymbol{\Gamma}_{NI}(t)) \end{aligned} \tag{A.3}$$

At each iteration, the algorithm runs L Kalman filters independently in order to estimate the $\boldsymbol{x}^{\mathrm{i}}(1{:}T)$, for i=1,…, L. Given all the $\boldsymbol{x}^{\mathrm{i}}(1{:}T)$, we aim to infer the mean and variance of the $N_E(1{:}T)$ and $N_I(1{:}T)$. The algorithm iterates using the estimated mean and variance of these SIs.

Appendix II. Inference of statistical parameters

We run Kalman filters (see details in Appendix 4 of [1]) for L trials to compute sufficient statistics required for EM algorithm.

We derive EM for our problem as described below.

$$
\begin{aligned}
&E_{p(N_E,N_I/Y)}\left\{\sum_i \log\left(p(y^i, X^i/\hat{\theta})\right)/Y,\theta\right\} = \\
&\sum_i\sum_{t=1}^{T} E_{p(N_E,N_I/Y)}\left\{-\frac{1}{2}\log\sigma_\varepsilon^2 + \left(y^i(t)-V^i(t)\right)^H \left(\sigma_\varepsilon^2\right)^{-1}\left(y^i(t)-V^i(t)\right)\right\} + \\
&\sum_i\sum_{t=1}^{T} E_{p(N_E,N_I/Y)}\left\{-\frac{1}{2}\log\Gamma_{NE}(t) + \left(\mathrm{N}_E^i(t)-\mu_{NE}(t)\right)^H \left(\Gamma_{NE}(t)\right)^{-1}\left(\mathrm{N}_E^i(t)-\mu_{NE}(t)\right)\right\} + \\
&\sum_i\sum_{t=1}^{T} E_{p(N_E,N_I/Y)}\left\{-\frac{1}{2}\log\Gamma_{NI}(t) + \left(\mathrm{N}_I^i(t)-\mu_{NI}(t)\right)^H \left(\Gamma_{NI}(t)\right)^{-1}\left(\mathrm{N}_I^i(t)-\mu_{NI}(t)\right)\right\}
\end{aligned}
\tag{A.4}
$$

Note that we aim to estimate a common SI for all given trials, where the dynamics of each trial produce their own synaptic perturbation (mean and variance of SIs underlying each single trial) as expressed below.

$$
\begin{aligned}
&N_E^i(t) = \mu_{NE}^i(t) + \sqrt{\Gamma_{NE}^i(t)}\,\xi_E^i(t),\ \xi_E^i(t) = Gauss(0,1) \\
&N_I^i(t) = \mu_{NI}^i(t) + \sqrt{\Gamma_{NI}^i(t)}\,\xi_I^i(t),\ \xi_I^i(t) = Gauss(0,1)
\end{aligned}
\tag{A.5}
$$

The terms $\mu^i{}_{NE}$, $\mu^i{}_{NI}$, $\Gamma^i{}_{NE}$, and $\Gamma^i{}_{NI}$ indicate the conditional mean (μ) and variance (Γ) of the SI given the observation of the i^{th} trial, namely, $\mu_{N_{E,I}}^i = E\{\mathrm{N}_{E,I}^i(t)\,|\,Y^i\}$ and $\Gamma_{N_{E,I}}^i = \mathrm{var}\{\mathrm{N}_{E,I}^i(t)\,|\,Y^i\}$. Note that we use (A.5) to simply express the conditional probabilities of SIs in (A.4).

By taking the derivative of (A.4) with respect to μ_N and Γ_N (for both excitatory and inhibitory inputs), the EM algorithm results in estimating the statistical parameters of the common SIs as follows:

$$
\begin{aligned}
&\mu_{NE}(t) = \frac{1}{L}\sum_{i=1}^{L} \mu^i{}_{NE}(t) \\
&\Gamma_{NE}(t) = \frac{1}{L}\sum_{i=1}^{L} \{\Gamma^i{}_{NE}(t) + (\mu_{NE}(t) - \mu^i{}_{NE}(t))^2\}
\end{aligned}
\tag{A.6}
$$

$$
\begin{aligned}
&\mu_{NI}(t) = \frac{1}{L}\sum_{i=1}^{L} \mu^i{}_{NI}(t) \\
&\Gamma_{NI}(t) = \frac{1}{L}\sum_{i=1}^{L} \{\Gamma^i{}_{NI}(t) + (\mu_{NI}(t) - \mu^i{}_{NI}(t))^2\}
\end{aligned}
\tag{A.7}
$$

To finish our EM section, we calculate, for excitatory and inhibitory SIs, the conditional mean of each trial, i.e., $\mu^i{}_{NE}$ and $\mu^i{}_{NI}$, and the trial average of the conditional variances, i.e., $\frac{1}{L}\sum_{i=1}^{L} \Gamma^i{}_{NE}$

and $\frac{1}{L}\sum_{i=1}^{L} \Gamma^{i}{}_{NI}$. Note that all sufficient (conditional) statistics of SCs have been calculated previously by KF.

Consider the dynamics of SCs in (1):

$$g_{E,I}(t+1) = (1 - dt/\tau_{E,I})g_{E,I}(t) + N_{E,I}(t) \qquad \text{(A.8)}$$

The mean and variance of SCs can be calculated as follows (see [2] for more details):

$$\begin{aligned} E\{g_{E,I}(t)\} &= (1 - dt/\tau)E\{g_{E,I}(t-1)\} + E\{N_{E,I}(t-1)\} \\ \mathrm{var}\{g_{E,I}(t)\} &= (1 - 2dt/\tau)\,\mathrm{var}\{g_{E,I}(t-1)\} + \mathrm{var}\{N(t-1)\} \end{aligned} \quad . \qquad \text{(A.9)}$$

Using the generic form of (A.9) for our problem, we have

$$\begin{aligned} \mu^{i}{}_{NE}(t) &= E\{\mathrm{g}_E^i(t+1) \,|\, Y^i\} - (1 - dt/\tau_E)\,\mathrm{E}\{\mathrm{g}_E^i(t) \,|\, Y^i\} \\ \mu^{i}{}_{NI}(t) &= E\{\mathrm{g}_I^i(t+1) \,|\, Y^i\} - (1 - dt/\tau_I)\,\mathrm{E}\{\mathrm{g}_I^i(t) \,|\, Y^i\} \\ \frac{1}{L}\sum_{i=1}^{L} \Gamma^{i}{}_{NE}(t) &\approx \frac{1}{L}\sum_{i=1}^{L} \left[\mathrm{var}\{\mathrm{g}_E^i(t+1) \,|\, Y^i\} - (1 - 2dt/\tau_E)\ \mathrm{var}\{\mathrm{g}_E^i(t) \,|\, Y^i\}\right] \\ \frac{1}{L}\sum_{i=1}^{L} \Gamma^{i}{}_{NI}(t) &\approx \frac{1}{L}\sum_{i=1}^{L} \left[\mathrm{var}\{\mathrm{g}_I^i(t+1) \,|\, Y^i\} - (1 - 2dt/\tau_I)\ \mathrm{var}\{\mathrm{g}_I^i(t) \,|\, Y^i\}\right] \end{aligned} \qquad \text{(A.10)}$$

The approximations in the previous two equations become more accurate if the number of trials L is large. Again, note that the statistics on the right-hand sides of these equations are calculated by KF.

References

[1] Lankarany, M., Zhu, W. P., Swamy, M. N., and Toyoizumi, T. (2013). Inferring trial-to-trial excitatory and inhibitory synaptic inputs from membrane potential using Gaussian mixture Kalman filtering. Front. Comput. Neurosci. 7, 109. doi: 10.3389/fncom.2013.00109

[2] Gillespie, D. T. (1996). The mathematics of Brownian Motion and Johnson noise. Am. J. Phys. 64, 225–240.